MIMO Cube Decoder

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***Abstract*—An insight on the lattice decoder for flat-fading multiple antenna wireless communications systems is presented in this paper. In particular, we show that by formulating the decoding problem as a bounded-error subset selection, the resultant decoder finds the nearest lattice point to the received signal vector such that the search is bounded inside a hypercube centered at the received vector. The dimensions and orientation of the hypercube can be adjusted based on the diversity of the channel in order to improve its performance. The search for the nearest codeword to the received signal vector is solved by modeling the problem as an Integer Program (IP). Simulation shows that the proposed decoder is inferior to the Sphere Decoder (SD) by about 1-dB while its complexity is superior to the Sphere Decoder at very low signal to noise ratio.**

I. Introduction

Searching for the nearest lattice point to a given point in multidimensional lattices arises in Multi-Input Multi-Output (MIMO) wireless communication systems, e.g., Spatial Multiplexing (SM) and Space-Time (ST) Codes. When the contaminating noise is Gaussian, the optimum receiver, the Maximum Likelihood (ML), is the minimum distance receiver which leads to exhaustive search across all lattice points. Unfortunately, with MIMO systems the lattice size grows exponentially with the number of transmit antennas and the ML solution would be infeasible. In particular, consider the complex-valued baseband MIMO model in flat fading channels with M transmit and N received antennas. Let the $N \times 1$ received signal vector $\grave{y}$,

$$\grave{\mathbf{y}} = \grave{\mathbf{H}}\grave{\mathbf{x}} + \grave{\mathbf{w}} \quad (1)$$

with the transmitted signal vector $\grave{\mathbf{x}} \in \mathbb{Z}_c^M$ whose elements are drawn from q-QAM constellation and $\mathbb{Z}_c$ is the set of complex integers, the $N \times M$ channel matrix $\grave{\mathbf{H}}$ whose elements h_{ij} represent the Rayleigh complex flat fading gain from transmitter j to receiver i with $h_{ij} \sim \mathcal{CN}(0,1)$. In this paper, it is assumed that channel realization is known to the receiver through preamble and/or pilot signals, and $N \geq M$. The $N \times 1$ complex noise vector $\grave{\mathbf{w}}$ has independent complex Gaussian elements with variance σ^2 per dimension. Throughout the paper, we will consider the real model of (1)

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w} \quad (2)$$

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Define $m = 2M, n = 2N$, then $\mathbf{y} = [\mathcal{R}(\grave{\mathbf{y}})\ \mathcal{I}(\grave{\mathbf{y}})]^T \in \mathbb{R}^n$, $\mathbf{x} = [\mathcal{R}(\grave{\mathbf{x}})\ \mathcal{I}(\grave{\mathbf{x}})]^T \in \mathbb{Z}^m$, $\mathbf{w} = [\mathcal{R}(\grave{\mathbf{w}})\ \mathcal{I}(\grave{\mathbf{w}})]^T \in \mathbb{R}^n$, and

$$\mathbf{H} = \begin{pmatrix} \mathcal{R}(\grave{\mathbf{H}}) & -\mathcal{I}(\grave{\mathbf{H}}) \\ \mathcal{I}(\grave{\mathbf{H}}) & \mathcal{R}(\grave{\mathbf{H}}) \end{pmatrix} \in \mathbb{R}^{n \times m},$$

where $\mathcal{R}(.)$ and $\mathcal{I}(.)$ are the real and imaginary parts, respectively. ML solution finds the symbol estimate $\hat{\mathbf{x}}_{\mathbf{ML}}$ that minimizes the 2-norm of the residual error. In particular

$$\hat{\mathbf{x}}_{\mathbf{ML}} = arg \min_{\mathbf{x} \subset \Lambda} \parallel \mathbf{y} - \mathbf{H}\mathbf{x} \parallel_2 \quad (3)$$

where Λ is the lattice whose points represent all possible codewords at the transmitter and $\mathbb{Z}^m$ is the set of integers of dimension m. The coordinates of the lattice points are all integers whose elements are drawn from the $L = \log_2(q)$-PAM derived from the q-QAM constellation. According to (2), ML solution leads to solving integer least-squares problem which is, in general, NP-hard [1]. Moreover, the number of lattice points in a given lattice Λ can be extremely large even for a reasonable number of transmit antennas. In SM for example, if $M = 4$ and 16-QAM constellation is used, then the number of all possible lattice points is 16^4 codewords. Another difficulty comes from the fact that the elements of $\hat{\mathbf{x}}$ are restricted to be integers and can only take values from the corresponding L-PAM.

Approximate solutions to (3) include the Zero Forcing (ZF) and MMSE solutions, V-BLAST which is a Nulling and Canceling technique with or without optimal ordering [2], [3]. Exact methods with reduced complexity is found for special orthogonal Space-Time Block Codes (STBC) [4], [5]. The Sphere Decoder (SD) of Fincke and Pohst searches for the lattice point inside a sphere centered at the received vector [6]–[8]. The SD achieves near-optimum performance but its complexity is a function of the SNR; a major drawback of the sphere decoder is that its average complexity is high for low signal to noise ratio and decreases with high SNR; this makes it difficult to use for low SNR environments [9].

The contribution of this paper can be summarized in the following two points. First, we show that by reformulating the decoding problem as a bounded-error subset selection, the resultant decoder finds the nearest lattice point to the received vector inside a hypercube centered at that point, hence the name ”Cube Decoder” (CD). We will show that the dimensions of the hypercube depends on channel diversity. In particular, for diverse channel, the channel matrix is well-conditioned and the cube will not be skewed much. On the other hand, for non-diverse channel, the channel matrix is ill-conditioned and the cube will be highly skewed. The second contribution, we show by simulation that the complexity of the proposed