Regularized MIMO Decoders

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***Abstract*—In the Multi Input Multi Output (MIMO) antenna system, it is known that the Linear Minimum Mean Squared Error (MMSE) receiver is equivalent to Tikhonov regularization. Given that, we develop a family of generalized receivers based on regularization with different penalty functions that penalize the received symbols outside the convex hull of the modulating constellation. For illustration purposes we consider two types of penalty functions, the deadzone and infinity norm penalty functions. The proposed decoders have low complexity and can be implemented efficiently using convex optimization algorithms. Simulation results show that the proposed receivers outperform the MMSE receiver by as high as 5-dB at low Signal to Noise Ratio (SNR).**

***Index Terms*—Multiple antenna systems, spatial multiplexing, lattice problems, wireless communications, regularization.**

I. Introduction

In Spatial Multiplexing (SM) scenario of the Multi Input Multi Output (MIMO) flat fading wireless communication system with m transmit and n receive antennas, the relation between the transmitted and the received signal can be described as follows

$$\mathbf{y} = \sqrt{\frac{\rho}{m}}\mathbf{H}\mathbf{x} + \mathbf{w} \quad (1)$$

where ρ is the expected value of the Signal to Noise Ratio (SNR) at each receive antenna, $\mathbf{x}$ is the $m \times 1$ transmitted vector whose elements are complex symbols drown form the normalized M-QAM constellation with $\mathbf{E}(\mathbf{x}\mathbf{x}^{\mathbf{T}}) = \mathbf{I}$, where M is the constellation order and $\mathbf{I}$ is the $m \times m$ identity matrix. $\mathbf{y}$ is the $n \times 1$ received vector, $\mathbf{H}$ is the $n \times m$ channel matrix with $n \geq m$, whose elements represent the i.i.d. flat fading channel gains $h_{ij} \sim \mathcal{CN}(0, \sigma_h^2)$. Without loss of generality, $\mathbf{w}$ is $n \times 1$, i.i.d. zero mean complex white Gaussian noise, uncorrelated with the transmitted symbols, with $w_i \sim \mathcal{CN}(0, 1)$. For the Gaussian noise scenario, the optimum decoder is the Maximum Likelihood decoder which finds the most likely input vector $\mathbf{x_{ml}}$ according to

$$\mathbf{x_{ml}} = \mathbf{arg} \min_{\mathbf{x} \in \Lambda} \|\mathbf{y} - \sqrt{\rho/m}\mathbf{H}\mathbf{x}\|^2 \quad (2)$$

where $\mathbf{\Lambda}$ is the lattice whose points represent all possible combinations of $\mathbf{x}$. The problem (2) is NP-hard in general that can only be exactly solved by exhaustive search over all possible M^m vector combinations; where its complexity in this case grows exponentially with the problem size [1]. This is due to the discrete nature of the lattice $\mathbf{\Lambda}$; nearest lattice point search algorithms can be used to solve (2) approximately [2], [3].

Hence, linear decoders have been used to obtain an approximate solution with lower complexity. The simplest linear decoder is the decorrelator which is known as the Zero-Forcing (ZF) decoder in which the constraint $\mathbf{x} \in \mathbf{\Lambda}$ is relaxed and the domain in this case is $\mathbb{R}^n$. The zero-forcing decoder inverts the channel in order to cancel spatial interference, in particular $\mathbf{x_{zf}} = \mathbf{G_{zf}}\ \mathbf{y}$ with

$$\mathbf{G_{zf}} = \sqrt{m/\rho}(\mathbf{H}^*\mathbf{H})^{-\mathbf{1}}\mathbf{H}^* \quad (3)$$

is the pseudo-inverse of the channel [4]. Although the zero forcing solution completely cancels out spatial interference, it has the disadvantage of enhancing the noise, especially if the channel matrix is ill-conditioned; in such case, small eigen values amplify the contaminating noise. In an effort to reduce noise enhancement, the Linear Minimum Mean Squared Error (MMSE) decoder is used to strike a balance between interference cancelation and noise enhancement [5]. The MMSE decoder finds the solution $\mathbf{x_{mmse}} = \mathbf{G_{mmse}}\ \mathbf{y}$ where

$$\mathbf{G_{mmse}} = \mathbf{arg} \min_{\mathbf{x}} \mathbf{E}\|\mathbf{G}\mathbf{y} - \mathbf{x}\|^{\mathbf{2}} \quad (4)$$

which has the analytical solution

$$\mathbf{G_{mmse}} = \sqrt{\frac{\rho}{m}}(\frac{\rho}{m}\mathbf{H}^*\mathbf{H} + \mathbf{I})^{-1}\mathbf{H}^* \quad (5)$$

It is clear that at high SNR the MMSE decoder converges to the ZF decoder, while at low SNR the MMSE decoder prevents noise amplification by improving small eigen values before matrix inversion. Hence, the MMSE decoder reduces noise enhancement at the expense of complete interference cancelation. Since the transmitted symbols are drawn from a specific constellation with certain alphabet, a slicing operation is required as a post-processing operation for both the zero forcing and the MMSE decoders over the transmitted constellation. Successive interference cancelation receivers such as the Bell Laboratories Layered Space-Time (BLAST) receivers are among the suboptimal categories for solving (2) [6], [7]. Although the BLAST receiver, in its two forms; the diagonal (D-BLAST) and the vertical (V-BLAST) versions, outperform the MMSE and ZF decoders, they suffer from error propagation due to the its successive nature. A near optimal receiver is the Sphere Decoder (SD) which finds the nearest lattice point inside a hypersphere, with variable radius, centered at the received signal point (2) [3], [8]. The SD has the best performance among all previous receivers but its performance varies as a function of the SNR [9]. A reminiscent of the sphere decoder is the Cube Decoder (CD) which finds the nearest lattice point inside a hypercube. The

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