Regular and Irregular Multiple Serially-Concatenated Multiple-Parity-Check Codes for Wireless Applications

Marco Baldi, Giovanni Cancellieri, Franco Chiaraluce and Amedeo De Amicis

***Abstract*—Multiple Serially-Concatenated Multiple-Parity-Check (M-SC-MPC) codes are a class of structured Low-Density Parity-Check (LDPC) codes, characterized by very simple encoding, that we have recently introduced. This paper evidences how the design of M-SC-MPC codes can be optimized for their usage in wireless applications. For such purpose, we consider some Quasi-Cyclic LDPC codes included in the mobile WiMax standard, and compare their performance with that of M-SC-MPC codes having the same parameters. We also present a simple modification of the inner structure of M-SC-MPC codes that can help to improve their error correction performance by introducing irregularity in the parity-check matrix and increasing the length of local cycles in the associated Tanner graph. Our results show that regular and irregular M-SC-MPC codes, so obtained, can achieve very good performance and compare favorably with standard codes.**

***Index Terms*— Error Correcting Codes, Mobile WiMax, M-SC-MPC Codes, QC-LDPC Codes.**

I. Introduction

The scenario of forward error correction has dramatically changed in the last decades, thanks to the introduction of codes based on the so called "turbo principle", like turbo codes [1] and Low-Density Parity-Check (LDPC) codes [2], [3]. The turbo principle consists in an iterated exchange and updating of reliability values referred to each received bit. More generally, iterative soft decoding algorithms use, as input, the soft information from the channel in the form of *a priori* probabilities on the status of each received bit, and iteratively update such values based on the parity-check constraints imposed by the code. Hence, the algorithm can produce *extrinsic* messages, to be used as starting values for the next iteration, and *a posteriori* probabilities, that represent the updated reliability values [4].

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M. Baldi, G. Cancellieri, F. Chiaraluce and A. De Amicis are with the Polytechnic University of Marche, Ancona, Italy (e-mail: {m.baldi; g.cancellieri; f.chiaraluce; a.deamicis}@univpm.it).

LDPC codes, in particular, have been proved to be able to approach the ultimate channel capacity limits [5], and are experiencing an increasing diffusion in many telecommunication standards and practical applications.

Very good LDPC codes can be obtained through a constrained random design of the parity-check matrix, but this, together with the need to adopt large codes for achieving good performance, can yield difficulties in their hardware implementation. For this reason, an increasing interest has been devoted to *structured* LDPC codes, that have characteristic matrices with a very simple inner structure, in such a way as to facilitate their implementation.

Structured LDPC codes can be found in several recommendations and practical applications, as, for example, the DVB-S2 [6] and the IEEE 802.16e (or Mobile WiMax) standard [7]. A very important class of structured LDPC codes is represented by Quasi-Cyclic (QC) LDPC codes, having parity-check and generator matrices formed by circulant blocks [8]. This form of the characteristic matrices allows efficient encoding and decoding, without penalizing the code performance. However, the design of QC-LDPC codes is *block-wise* oriented, thus yielding some constraints on the choice of the code parameters.

As an alternative, we have recently proposed Multiple Serially Concatenated Multiple Parity-Check (M-SC-MPC) codes, that allow to design structured LDPC codes without loosing flexibility in the choice of their parameters [9]. M-SC-MPC codes are obtained as the serial concatenation of very simple component codes, named MPC codes, that are a generalization of Single Parity-Check (SCP) codes. This allows to design a concatenated encoder based on very simple circuits. The parity-check matrix of an M-SC-MPC is lower triangular and has columns with almost uniform Hamming weights. Under suitable hypotheses, such parity-check matrix can be sparse and free of short cycles; so, M-SC-MPC can be seen as LDPC codes and their decoding can be accomplished through efficient LDPC decoding algorithms based on the *belief propagation* principle.

In this paper, we show how M-SC-MPC codes can be designed for practical applications and compare them with standard QC-LDPC codes.

We refer to the IEEE 802.16e standard and give some design examples of M-SC-MPC codes with the same parameters as standard QC-LDPC codes, in order to compare their simulated error correction performance. Furthermore, we present a very simple technique to introduce some form of irregularity in the parity-check matrix of an M-SC-MPC code. Irregular LDPC codes, in fact, have been proved to be better than regular ones, especially for low code rates [10]. For this reason, we propose a solution to design irregular M-SC-MPC matrices, that can also have longer local cycles in the associated Tanner graph. Simulations prove that irregular M-SC-MPC codes can actually outperform almost regular ones.

The paper is organized as follows. Section 2 describes the QC-LDPC codes adopted in the IEEE 802.16e standard. Section 3 summarizes the characteristics of our recently proposed M-SC-MPC codes and introduce a solution for designing irregular M-SC-MPC codes. Section 4 reports some design examples of regular and irregular M-SC-MPC codes. Section 5 describes the results of numerical simulations of the designed codes and their performance comparison with standard QC-LDPC codes. Finally, Section 6 concludes the paper.

I. QC-LDPC Codes in Mobile WiMax

The IEEE 802.16e Mobile Wireless MAN standard, approved in 2005 [7], includes, as an option, the possibility of adopting LDPC codes for forward error correction (FEC). As common in wireless applications, the FEC scheme must be able to adapt itself against variable channel conditions, providing different solutions as tradeoffs between error correction capability and spectral efficiency.

For this reason, a family of channel codes with different rates must be provided. In addition, all standards in the IEEE 802 family deal with packet switched networks having variable length blocks; so, flexibility in the block length is a mandatory requirement for channel codes to be used in these systems.

In order to fulfill such needs, the IEEE 802.16e standard adopts a family of QC-LDPC codes having variable code rate (R = 1/2, 2/3, 3/4, 5/6), and length (n) ranging between 576 and 2304 bits by a step of 96 bits. These codes are used in conjunction with the following modulation schemes: QPSK, 16-QAM and 64-QAM.

Standard QC-LDPC codes have parity-check matrices formed by $z\times z$ circulant permutation blocks or null blocks. The parity-check matrix of each standard QC-LDPC code can be represented as shown in (1), where $\mathbf{I}_{p(i,j)}$, $0 \le i \le r_b - 1$, $0 \le j \le n_b - 1$, is the $z\times z$ circulant permutation matrix corresponding to the cyclic shift $p(i, j) \in [0; z - 1]$.

$$\mathbf{H} = \begin{bmatrix} \mathbf{I}_{p(0,0)} & \mathbf{I}_{p(0,1)} & \cdots & \mathbf{I}_{p(0,n_b-1)} \\ \mathbf{I}_{p(1,0)} & \mathbf{I}_{p(1,1)} & \cdots & \mathbf{I}_{p(1,n_b-1)} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{I}_{p(r_b-1,0)} & \mathbf{I}_{p(r_b-1,1)} & \cdots & \mathbf{I}_{p(r_b-1,n_b-1)} \end{bmatrix}. \quad (1)$$

This allows to obtain a synthetic representation of the parity-check matrices: if each null block is conventionally associated to $\mathbf{I}_{-1}$, matrix $\mathbf{H}$ in (1) can be represented in an alternative form through a *base matrix*, $\mathbf{H}_{\mathbf{bm}}$, having size $r_b\times n_b$, that contains the $p(i, j)$ values associated to each block. For the IEEE 802.16e standard codes, it is $n_b = 24$, while r_b varies according with the code rate.

Two examples of $\mathbf{H}_{\mathbf{bm}}$ are reported in (2), for the rate 3/4 "B" standard code, and in (3), for the rate 5/6 standard code. From the structure of the base matrix we notice that, for both rates, the parity-check matrices of the codes have a dual-diagonal form, that is, contain two diagonals of symbols 1 in their rightmost part. This property can facilitate encoding, when accomplished through the parity-check matrix [11], without yielding the penalization in minimum distance that would be due to the inclusion of identity blocks. The same also occurs for the other base matrices provided by the

$$\mathbf{H}_{\mathbf{bm}}^{R34B} = \begin{bmatrix} -1 & 81 & -1 & 28 & -1 & -1 & 14 & 25 & 17 & -1 & -1 & 85 & 29 & 52 & 78 & 95 & 22 & 92 & 0 & 0 & -1 & -1 & -1 & -1 \\ 42 & -1 & 14 & 68 & 32 & -1 & -1 & -1 & -1 & 70 & 43 & 11 & 36 & 40 & 33 & 57 & 38 & 24 & -1 & 0 & 0 & -1 & -1 & -1 \\ -1 & -1 & 20 & -1 & -1 & 63 & 39 & -1 & 70 & 67 & -1 & 38 & 4 & 72 & 47 & 29 & 60 & 5 & 80 & -1 & 0 & 0 & -1 & -1 \\ 64 & 2 & -1 & -1 & 63 & -1 & -1 & 3 & 51 & -1 & 81 & 15 & 94 & 9 & 85 & 36 & 14 & 19 & -1 & -1 & -1 & 0 & 0 & -1 \\ -1 & 53 & 60 & 80 & -1 & 26 & 75 & -1 & -1 & -1 & -1 & 86 & 77 & 1 & 3 & 72 & 60 & 25 & -1 & -1 & -1 & -1 & 0 & 0 \\ 77 & -1 & -1 & -1 & 15 & 28 & -1 & 35 & -1 & 72 & 30 & 68 & 85 & 84 & 26 & 64 & 11 & 89 & 0 & -1 & -1 & -1 & -1 & 0 \end{bmatrix} \quad (2)$$

$$\mathbf{H}_{\mathbf{bm}}^{R56} = \begin{bmatrix} 1 & 25 & 55 & -1 & 47 & 4 & -1 & 91 & 84 & 8 & 86 & 52 & 82 & 33 & 5 & 0 & 36 & 20 & 4 & 77 & 80 & 0 & -1 & -1 \\ -1 & 6 & -1 & 36 & 40 & 47 & 12 & 79 & 47 & -1 & 41 & 21 & 12 & 71 & 14 & 72 & 0 & 44 & 49 & 0 & 0 & 0 & 0 & -1 \\ 51 & 81 & 83 & 4 & 67 & -1 & 21 & -1 & 31 & 24 & 91 & 61 & 81 & 9 & 86 & 78 & 60 & 88 & 67 & 15 & -1 & -1 & 0 & 0 \\ 50 & -1 & 50 & 15 & -1 & 36 & 13 & 10 & 11 & 20 & 53 & 90 & 29 & 92 & 57 & 30 & 84 & 92 & 11 & 66 & 80 & -1 & -1 & 0 \end{bmatrix} \quad (3)$$

It follows from its definition that an MPC code can be encoded by using a circuit as that reported in Fig. 2 for the i-th component of the serial concatenation.

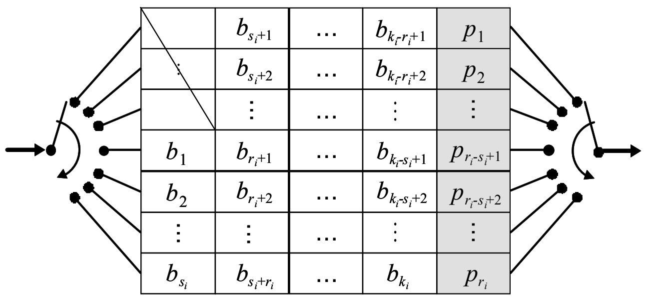

Fig. 2. Encoder for the i-th MPC component code.

This circuit is formed by a rectangular matrix having r_i rows and $\lceil k_i/r_i \rceil + 1$ columns, where $\lceil x \rceil$ represents the *ceiling function*, that gives the smallest integer greater than or equal to x. The white cells in the matrix are filled with the information word input to the i-th MPC code, in column-wise order, from top left to bottom right. The first $r_i - s_i$ cells, with $s_i = k_i \bmod r_i$, are unused, whereas used cells are denoted in the figure by their corresponding information bit index. When the j-th row is full, $j = 1 \ldots r_i$, the parity bit p_j is calculated, by XORing the elements of the row, and its value is stored in the last column, at the same row (grey cells in the figure). When all the parity bits have been calculated and the last column is full, the encoder outputs the codeword by reading its content in the same order used for the input, but including also the parity bits at the end of the codeword.

It follows from the definition and the encoder structure of an MPC code that its parity-check matrix has a very simple form, almost coincident with a single row of $r_i \times r_i$ identity matrices. Fig. 3 shows its form for the i-th MPC component code in the serial concatenation. In the figure, diagonals represent symbols 1, whereas all other symbols are null.

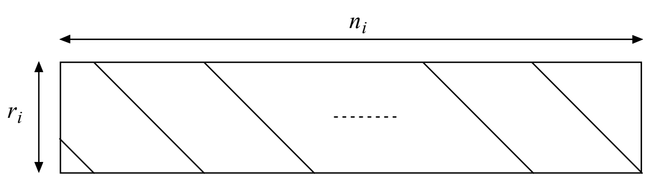

Fig. 3. Parity-check matrix of the i-th MPC component code.

The component parity-check matrices (having the form shown in Fig. 3) can be combined to obtain a valid parity-check matrix for the serially concatenated code, that has the form shown in Fig. 4 (for the case with $M = 3$). Such matrix has $r = \sum_{i=1}^{M} r_i$ rows, and the maximum Hamming weight of its columns is M. So, it is immediate to observe that, for large r_i values, the parity-check matrix is sparse, and M-SC-MPC codes can be seen as LDPC codes.

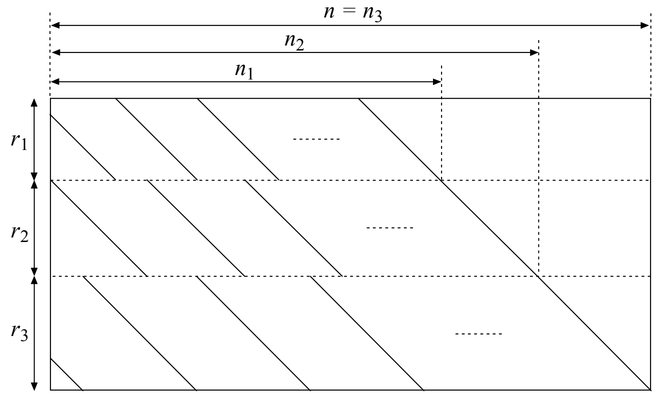

Fig. 4. Parity-check matrix of an M-SC-MPC code with $M = 3$.

Even more noticeably, it can be proved that, for distinct, coprime and increasingly ordered r_i's, the matrix is free of length-4 cycles when the following condition is satisfied [9]:

$$n \le n_{\max} = r_1 r_2 + \sum_{i=2}^{M} r_i \; ; \tag{8}$$

so, efficient belief propagation algorithms can be adopted for decoding M-SC-MPC codes as LDPC codes.

Condition (8) allows great flexibility in the code design, since the value of $n_{\max}$ is rather large (for common choices of the r_i values) and each value of length $n \le n_{\max}$ is theoretically feasible. Moreover, the very simple structure of the parity-check matrix, together with its lower triangular form, allows easy encoding when accomplished through standard techniques, like *back substitution*, instead of adopting concatenated MPC encoders.

B. Nodes degree distributions

From the definition of M-SC-MPC codes, and from the structure of their parity-check matrices, it follows that such codes are almost regular. In other terms, their associated Tanner graphs have almost constant variable and check nodes degrees. In order to represent variable and check nodes degree distributions, we can refer to the notation introduced in [10]. According with that notation, an irregular bipartite graph with maximum variable nodes degree $\overline{d_v}$ and maximum check nodes degree $\overline{d_c}$ is specified by two sequences, ($\lambda_1, \ldots, \lambda_{\overline{d_v}}$) and ($\rho_1, \ldots, \rho_{\overline{d_c}}$), such that λ_i (ρ_i) is the fraction of edges connected to variable (check) nodes with degree i. These two sequences can be used as the coefficients of two polynomials, $\lambda(x)$ and $\rho(x)$, describing the edge degree distributions:

$$\begin{cases} \lambda(x) = \sum_{i=1}^{\overline{d_v}} \lambda_i x^{i-1}; \\ \rho(x) = \sum_{i=1}^{\overline{d_c}} \rho_i x^{i-1}. \end{cases} \tag{9}$$

TABLE II
PARAMETERS OF THE SIMULATED CODES WITH LENGTH n = 1632

Code	R	Type	$[r_1, \ldots, r_M]$	$[h_1, \ldots, h_M]$	$v(x)$	$c(x)$
1	1/2	Q	-	-	$0.208x^5 + 0.333x^2 + 0.458x$	$0.333x^6 + 0.667x^5$
2	1/2	M	[153, 155, 159, 167, 182]	-	$0.594x^4 + 0.095x^3 + 0.097x^2 + 0.102x + 0.112$	$0.369x^8 + 0.301x^7 + 0.205x^6 + 0.125x^5$
3	1/2	M	[87, 89, 93, 101, 117, 149, 180]	-	$0.553x^6 + 0.055x^5 + 0.057x^4 + 0.062x^3 + 0.072x^2 + 0.091x + 0.110$	$0.203x^{11} + 0.327x^{10} + 0.217x^9 + 0.252x^8$
4	1/2	*i*M	[87, 89, 93, 101, 117, 149, 180]	[0, 4, 4, 4, 4, 3, 3]	$0.146x^5 + 0.275x^4 + 0.248x^3 + 0.058x^2 + 0.053x + 0.221$	$0.04x^{10} + 0.066x^9 + 0.328x^7 + 0.359x^6 + 0.206x^5$
5	2/3	Q	-	-	$0.667x^3 + 0.042x^2 + 0.292x$	$0.125x^{10} + 0.875x^9$
6	2/3	M	[113, 127, 149, 155]	-	$0.736x^3 + 0.078x^2 + 0.091x + 0.095$	$0.388x^{10} + 0.588x^9 + 0.024x^8$
7	2/3	M	[71, 83, 101, 127, 162]	-	$0.710x^4 + 0.051x^3 + 0.062x^2 + 0.078x + 0.099$	$0.042x^{16} + 0.088x^{15} + 0.147x^{14} + 0.061x^{13} + 0.131x^{12} + 0.134x^{11} + 0.121x^{10} + 0.276x^9$
8	2/3	*i*M	[71, 83, 101, 127, 162]	[4, 0, 3, 2, 0]	$0.292x^4 + 0.415x^3 + 0.116x^2 + 0.078x + 0.099$	$0.147x^{14} + 0.006x^{13} + 0.042x^{12} + 0.088x^{11} + 0.208x^{10} + 0.410x^9 + 0.099x^8$
9	3/4	Q	-	-	$0.292x^5 + 0.5x^2 + 0.208x$	$0.667x^{14} + 0.333x^{13}$
10	3/4	M	[59, 73, 113, 163]	-	$0.786x^3 + 0.045x^2 + 0.069x + 0.099$	$0.108x^{21} + 0.037x^{20} + 0.103x^{18} + 0.076x^{17} + 0.277x^{12} + 0.005x^{10} + 0.395x^9$
11	3/4	M	[73, 75, 79, 87, 94]	-	$0.795x^4 + 0.046x^3 + 0.048x^2 + 0.053x + 0.057$	$0.125x^{18} + 0.618x^{17} + 0.257x^{16}$
12	3/4	*i*M	[73, 75, 79, 87, 94]	[4, 0, 3, 2, 0]	$0.423x^4 + 0.359x^3 + 0.107x^2 + 0.053x + 0.058$	$0.054x^{18} + 0.213x^{17} + 0.147x^{16} + 0.216x^{15} + 0.191x^{14} + 0.137x^{13} + 0.042x^{12}$
13	5/6	Q	-	-	$0.458x^3 + 0.417x^2 + 0.125x$	x^{19}
14	5/6	M	[37, 53, 73, 109]	-	$0.856x^3 + 0.033x^2 + 0.045x + 0.067$	$0.103x^{37} + 0.033x^{36} + 0.07x^{27} + 0.125x^{26} + 0.232x^{20} + 0.037x^{19} + 0.390x^{14} + 0.011x^{13}$
15	5/6	M	[43, 45, 49, 57, 78]	-	$0.860x^4 + 0.028x^3 + 0.03x^2 + 0.035x + 0.048$	$0.129x^{32} + 0.195x^{31} + 0.099x^{30} + 0.081x^{29} + 0.055x^{27} + 0.154x^{26} + 0.265x^{20} + 0.022x^{19}$
16	5/6	*i*M	[43, 45, 49, 57, 78]	[12, 11, 0, 6, 4]	$0.156x^4 + 0.458x^3 + 0.286x^2 + 0.052x + 0.048$	$0.993x^{30} + 0.081x^{29} + 0.239x^{21} + 0.235x^{20} + 0.059x^{19} + 0.265x^{16} + 0.022x^{15}$

minimum distance.

III. CODE EXAMPLES

In this section, we provide some examples of codes with the aim to compare M-SC-MPC codes and their irregular versions with IEEE 802.16e standard QC-LDPC codes. For this purpose, we consider codes having all values of rate specified by the standard (R = 1/2, 2/3, 3/4 and 5/6) and length n = 1632. We have designed some M-SC-MPC codes and irregular M-SC-MPC codes with exactly the same length and rate.

Table 2 reports the parameters of the codes considered in our examples, that are of three types: QC-LDPC codes compliant with the IEEE 802.16e standard (Q), M-SC-MPC codes (M) and irregular M-SC-MPC codes (*i*M). In the latter case, the table also reports the number (h_j) of blocks canceled in each matrix layer (that is, the number of columns switched off in each component encoder). Such values, that form what we call a *nulling pattern*, have been found on a heuristic basis; so, margins may exist for their further optimization.

More precisely, for a given nulling pattern, the choice of the blocks to cancel in each layer has been made through a constrained random search, based on two criteria: i) avoiding the appearance of very low weight columns in the non-triangular part of the matrix (denoted by $d_v(i)$ the weight of the i-th column, we fixed $d_v(i) \geq 3$, $i \in [1; n_1]$) and ii) increasing the length of local cycles. For all code rates, different values of M have been considered for the design of M-SC-MPC codes. This allows to highlight the effect of varying the number of components codes. Codes with lower M have lower column degree of their parity-check matrices and smaller minimum distance. These two facts yield a rather good waterfall performance, that usually begins at lower signal-to-noise ratio with respect to codes with higher M. On the other hand, a small minimum distance produces rather high error floors, that cause a slope change in the error rate curves. When the number of component codes is increased, both the parity-check matrix column weight and the minimum distance are increased. This makes the waterfall begin at higher signal-to-noise ratios, but the error floor is lowered as well. So, the error rate curves for codes with low and high M tend to intersect at intermediate values of the signal-to-noise ratio. By applying block cancellations to codes with high M, the advantages of both low and high M can be joined, and codes can achieve good error rate performance both in the waterfall and in the error floor region. The simulation results of some specific cases will be discussed in the next section.

Table 2 also reports, for each code, the polynomials $v(x)$

standard QC-LDPC code (C_1) achieves very good performance, and its curves are the leftmost ones. The M-SC-MPC code with $M = 5$ (C_2) has a rather good performance, yielding a loss of about 0.2 dB with respect to the standard code. However, especially in its FER curve, an error floor effect is observed, that tends to deteriorate its performance for increasing signal-to-noise ratios.

The error floor can be mitigated by adopting a higher value of M: the M-SC-MPC code with $M = 7$ (C_3) exhibits a more favorable slope in its error rate curves for increasing signal-to-noise ratio. On the other hand, the adoption of such a high value of M gives a worse performance in the waterfall region, requiring higher signal-to-noise ratios for reaching the same error rate with respect to codes having lower M. Applying the cancellation of some blocks helps to mitigate such an effect: we observe that the irregular M-SC-MPC code (C_4), though still exploiting $M = 7$ component codes, has good performance in the waterfall region, with a slight improvement with respect to C_2. Moreover, in the error floor region, the performance of C_4 tends to further improve that of C_2, especially for the FER curve, in which the error floor effect is mitigated. So, the irregular M-SC-MPC code, obtained by canceling some blocks in the parity-check matrix of the M-SC-MPC code with $M = 7$, is able to join the advantages of both low and high M, and to achieve better performance both in the waterfall and in the error floor region. Its curves approach those of the standard code, with a loss of some fraction of dB with respect to them.

B. Codes with rate 2/3

The simulation results are quite similar for codes with rate 2/3: also for such code rate, irregular M-SC-MPC codes are able to achieve very good performance. Moreover, in this case, they can become even better than standard codes.

This can be observed in Fig. 6, where we notice that both the standard QC-LDPC code (C_5) and the M-SC-MPC code with $M = 4$ (C_6) show an error floor. On the contrary, the M-SC-MPC code with $M = 5$ (C_7) has no error floor, but its waterfall performance is worse than the others. The irregular M-SC-MPC code with $M = 5$ (C_8), instead, is able to improve the waterfall performance without loosing its curve slope in the error floor region.

C. Codes with rate 3/4

A similar conclusion can be drawn from the analysis of codes with rate 3/4, shown in Fig. 7: the M-SC-MPC code with low M (C_{10}) has good waterfall performance, but a rather high error floor. On the contrary, the M-SC-MPC code obtained by increasing M (C_{11}) has better error floor performance at the expense of the waterfall behavior. The adoption of an irregular M-SC-MPC code (C_{12}) allows to improve the error floor performance without loss in terms of error floor, so approaching the performance of the standard QC-LDPC code (C_9).

D. Codes with rate 5/6

For codes with rate 5/6, the difference among the considered design techniques is less evident. The simulation results, reported in Fig. 8, show that both the M-SC-MPC code with $M = 4$ (C_{14}) and the irregular M-SC-MPC code with $M = 5$ (C_{16}) are able to approach the performance of the standard code (C_{13}). The M-SC-MPC code with $M = 5$ (C_{15}) has instead a worse waterfall behavior. In this case, the usage of an irregular M-SC-MPC code does not give any significant improvement with respect to the classic M-SC-MPC solution.

V. Conclusion

In this paper, we have presented a design technique for codes intended for wireless applications. We have referred to QC-LDPC codes included in the IEEE 802.16e standard, and we have designed codes with the same parameters in order to compare their characteristics and performance with those of standard codes.

Our analysis has started from the M-SC-MPC codes design technique, we have recently proposed, and a new variant of it has been introduced, that is aimed at designing irregular codes. Both techniques are able to produce very good codes, with performance that can be comparable or even better with respect to standard codes.

We have shown that, in some cases, classic M-SC-MPC codes still have performance comparable with that of the new irregular M-SC-MPC codes. Our analysis brings to the conclusion that further margins of improvement may exist, that are probably related with the optimization of the nodes degree polynomials. This could be a point worth to be studied in future works.

Another interesting cue for future work is to deepen the investigation of the link between Quasi-Cyclic and M-SC-MPC codes, in such a way to assess whether the M-SC-MPC structure can also be adapted to design QC codes.

References

[1] C. Berrou, A. Glavieux, and P. Thitimajshima, *"Near Shannon limit error-correcting coding and decoding: Turbo-codes,"* in Proc. IEEE International Conference on Communications (ICC `93), Geneva, Switzerland, vol. 2, pp. 1064-1070, May 1993.

[2] R. G. Gallager, *"Low Density Parity Check Codes,"* Monograph, M.I.T. Press, 1963.

[3] T. Richardson and R. Urbanke, *"The capacity of low-density parity-check codes under message-passing decoding,"* IEEE Trans. Inform. Theory, vol. 47, no. 2, pp. 599–618, Feb. 2001.

[4] J. Hagenauer, E. Offer, and L. Papke, *"Iterative decoding of binary block and convolutional codes,"* IEEE Trans. Inform. Theory, vol. 42, no. 2, pp. 429–445, Mar. 1996.

[5] C. Sae-Young, G. Forney, T. Richardson, and R. Urbanke, *"On the design of low-density parity-check codes within 0.0045 dB of the Shannon limit,"* IEEE Commun. Lett., vol. 5, no. 2, pp. 58–60, Feb. 2001.

[6] ETSI EN 302 307 V1.1.1. *Digital Video Broadcasting (DVB); Second Generation Framing Structure, Channel Coding and Modulation Systems for Broadcasting, Interactive Services, News Gathering and Other Broadband Satellite Applications.* 2004.

[7] 802.16e 2005, IEEE Standard for Local and Metropolitan Area Networks - Part 16: *Air Interface for Fixed and Mobile Broadband Wireless Access Systems*, IEEE Std., Dec. 2005.