Reduction of Conducted Perturbations in DC-DC Voltage Converters by a Dual Randomized PWM Scheme

N. Boudjerda, M. Melit, B. Nekhoul, K. El khamlichi Drissi and K Kerroum

Abstract: **Randomized Pulse Width Modulation (RPWM) deals better than Deterministic PWM (DPWM) with Electro-Magnetic Compatibility (EMC) standards for conducted Electro-Magnetic Interferences (EMI). In this paper, we propose a dual RPWM scheme for DC-DC voltage converters: the buck converter and the full bridge converter. This scheme is based on the comparison of deterministic reference signals (one signal for the buck converter and two signals for the full bridge converter) to a single triangular carrier having two randomized parameters. By using directly the randomized parameters of the carrier, a mathematical model of the Power Spectral Density (PSD) of output voltage is developed for each converter. The EMC advantage of the proposed dual randomization scheme compared to the classical simple randomization schemes is clearly highlighted by the PSD analysis and confirmed by FFT (Fast Fourier Transform) analysis of the output voltage.**

Index terms: Electromagnetic compatibility, DC-DC converters, RPWM, power spectral density.

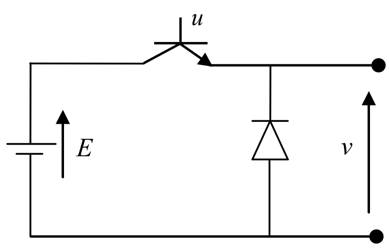
I. INTRODUCTION

Deterministic Pulse Width Modulation (DPWM) generates discrete harmonics with important magnitudes. The recent Electro-Magnetic Compatibility (EMC) standards impose more and more filtering effort in power converters [1]. In order to relax the filtering effort, the switching frequency is generally increased. However, this solution remains limited by the switching losses and the radiated electromagnetic interferences (EMI) generation [1]. One of the recent solutions is to use RPWM technique, which deals better with EMC standards by spreading the voltage spectrum in a large frequency range and reducing its amplitude [2-4]. Several works regarding this new control technique has been published lately, principally two randomization schemes are proposed; Randomized Carrier Frequency Modulation (RCFM) and Randomized Pulse Position Modulation (RPPM), for DC-DC conversion [4, 5] and for DC-AC conversion [2, 3, 5, 6]. Combinations of two randomized parameters have been also applied to the buck converter [1, 7] and to the three phase full bridge inverter [8].

In order to obtain a more spread spectrum with a significant reduction of its amplitude [9], we propose in this paper a combination of two simple RPWM schemes (RCFM and RPPM schemes) that we call RCFM-RPPM or dual RPWM scheme, for DC-DC voltage converters: the buck converter and the full bridge converter. The switching signals are generated by comparing a triangular carrier having two randomized parameters to deterministic reference signals (one reference signal for the buck converter and two reference signals for the full bridge converter). Generally the randomization is introduced directly into the switching signals and this isn't a simple task for converters needing more than one signal such as the full bridge converter [5, 6]. In the proposed scheme, the randomization is applied to the carrier rather than switching signals, which allows limiting the random parameters to those of the carrier only and thus facilitates the randomization [8, 9]. At first, we propose the modulating principle. Then a general analytical model of the voltage PSD is developed for the two converters, this model is expressed directly using the random parameters of the carrier. The particular cases (RCFM and RPPM schemes) can be deduced from the general model. The PSD analysis shows that the proposed dual RPWM scheme allows a better spread shape of PSD compared to the simple randomization schemes that is the desired EMC advantage. Finally, the FFT (Fast Fourier Transform) analysis of the voltage confirms this advantage.

II. MODULATING PRINCIPLE

The structures of the two converters under study are given in Fig. 1; the buck converter requires one switching function u and the full bridge converter requires two switching functions u_a and u_b.



a. Buck converter

Manuscript received November 07, 2007, and revised December 30, 2008.
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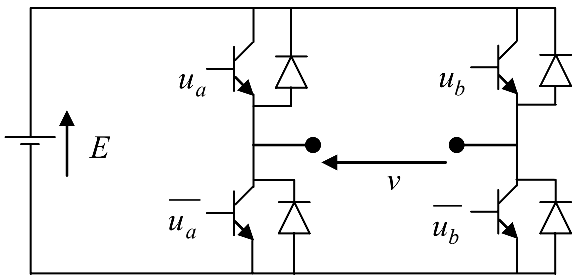


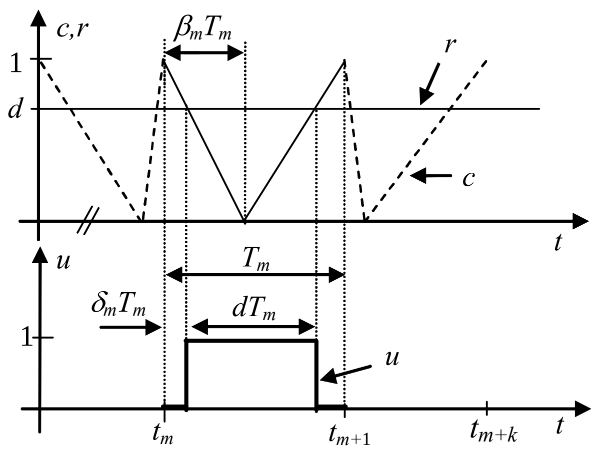
b. Full bridge converter

Fig.1. DC-DC voltage converters

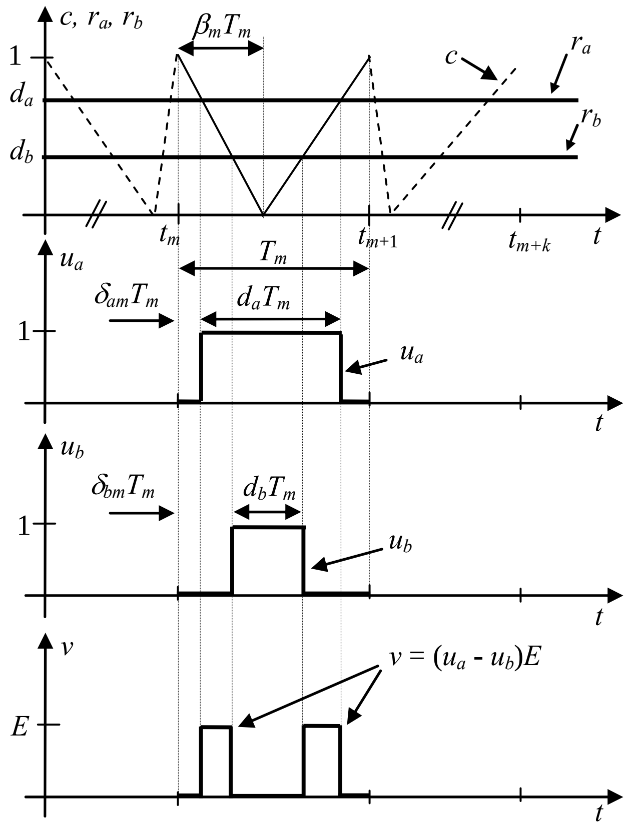
The modulating principle is illustrated in Fig.2:

- For the buck converter, the switching function u is obtained by comparing a deterministic reference signal r of magnitude d $(0 < d < 1)$ to a randomized triangular carrier c (Fig.2.a).
- For the full bridge converter, the two switching functions u_a and u_b are obtained by comparing two deterministic reference signals r_a and r_b of magnitudes d_a and d_b respectively, to a single randomized carrier c (Fig.2.b). Generally, the amplitudes d_a and d_b are taken as follows:

$$\begin{cases} 0 < d_a < 1 \\ 0 < d_b < 1 \\ d_a + d_b = 1 \end{cases} \quad (1)$$



a. Buck converter



b. Full bridge converter

Fig.2. Modulating principle

At low frequencies, the switching effects of power components are generally neglected [10 - 12], thus the output voltage v can be expressed in terms of the input voltage E and the switching functions u, u_a and u_b as follows:

- Buck converter:

$$v = uE \quad (2)$$

- Full bridge converter:

$$v = (u_a - u_b)E \quad (3)$$

Each of the switching functions (u, u_a or u_b) is completely characterized by three parameters (Fig.2): the switching period T, the duty cycle d and the delay report δ. In RPWM, these parameters should be randomized in a combined or a separated way. However, in industrial applications, the duty cycle d is generally deduced from a reference signal and allows the control of output voltage. Thus, only the switching period T (i.e. the period of the carrier) and the delay reports of the switching functions can be really randomized.

From (Fig.2.a), the delay report δ of the switching function u can be expressed as follows:

$$\delta = \beta(1 - d) \quad (4)$$

Where:

β : fall time report of the randomized carrier c, (Fig.2).
d: duty cycle of the switching signal, obtained by a fixed reference signal of amplitude d.

For the full bridge converter (Fig.1.b), the delay reports δ_a and δ_b (Fig.2.b) are obtained by using the two reference signals r_a and r_b of amplitudes d_a and d_b respectively in equation (4). Thus, a randomization of β in the interval [0, 1] gives a random delay report δ in the interval $[0,\ (1-d)]$: the resulting position of the corresponding switching function varies randomly from the beginning to the end of the switching period, $(\delta_{\min}=0 \text{ and } \delta_{\max}=1-d)$, (Fig.2.). Thus, for the two converters, the RPPM scheme is obtained by a triangular carrier with fixed period T and randomized fall time report β, (Fig.2). The particular case of Random Lead Lag Modulation (RLLM) is obtained by using two discrete random values of β with equal probability p_β $(\beta=0 \text{ or } \beta=1,\ p_\beta=0.5)$. We notice that the principal advantage of this particular scheme is the reduction of the switching losses [5].

RCFM scheme needs a carrier with randomized period T and fixed fall time report β. The randomization limits T_{min} and T_{max} of the period T are generally fixed around a mean value $\overline{T}$. For the buck converter, a saw tooth with a randomized period T is generally used (β = 0) and for the full bridge converter, the carrier is generally symmetrical (β = 0.5), with a randomized period T.

The proposed dual RPWM scheme (RCFM-RPPM) combines the two previous schemes; the two parameters of the carrier (T and β) are independently randomized in the intervals defined for the two simple RPWM schemes respectively, (RCFM and RPPM).
The resulting RPWM schemes for the two converters under study are summarized in Tab.1.

Tab.1. Resulting RPWM schemes

PWM Scheme	β	T
DPWM	fixed(*)	fixed
RPPM	randomized	fixed
RCFM	fixed(*)	randomized
RCFM-RPPM	randomized	randomized

$$\text{fixed}\ (*): \begin{Bmatrix} \text{Buck converter}: \beta = 0 \\ \text{Full bridge converter}: \beta = 0.5 \end{Bmatrix}$$

III. Power Spectral Density (PSD) of output voltage

The spectral analysis of output voltage can be performed either by Fast Fourier Transform (FFT) or by Power Spectral Density (PSD):

- FFT analysis: a τ length sample of the considered random signal is required for the FFT computation (the signal is assumed periodic of period τ). From a statistical point of view, the result is not exact; it depends on the time length τ. However in practice, several studies based on the FFT of random signals are performed in RPWM [2, 3].
- PSD analysis: the PSD (Power Spectral Density) is an exact statistical parameter of random signals (i.e. Fourier transform of the autocorrelation); it is particularly useful in information theory because it leads to exact statistical characteristics [12, 13]. However, to set a mathematical model of PSD isn't generally a simple task for all RPWM schemes and for complex structures of the converters such as the full bridge converter.

In this section, we develop a unified analytical model of PSD of output voltage for the two converters. First, this model is developed for RCFM-RPPM scheme and then the two simplified schemes (RPPM and RCFM) can be found as particular cases.

The PSD of a random signal $u(t)$ can be expressed as follows [13]:

$$W(f)=\lim_{\tau\to\infty}\frac{1}{\tau}E\left[\left|F(u_\tau(t))\right|^2\right] \tag{5}$$

Where:

$u_\tau(t)$: Expression of the signal during the time interval τ.

$F(u_\tau(t))$: Fourier transform of the signal sample $u_\tau(t)$.

$E\,[\,.\,]$: Statistical expectation.

For a random pulse signal, belonging to the class of Wide Sense Stationary (WSS) signals, the expression (5) leads to the expression (6), [4-9]:

$$W(f)=\lim_{N\to\infty}\frac{1}{\overline{T}}E\left[\sum_{k=-N}^{N}U_m(f)U_{m+k}^*(f)\right] \tag{6}$$

Where:

$\overline{T}$: Statistical mean of the switching period.
$U_m(f)$: Fourier transform of the pulse signal during an arbitrary switching period T_m.
$U_{m+k}^*(f)$: Complex conjugate of $U_{m+k}(f)$

The General expression (6) can be developed as follows [4-9]:

$$W(f)=\frac{1}{\overline{T}}\left\{E\left[\left|U_m(f)\right|^2\right]+2\,\text{Real}\left(\sum_{k=1}^{\infty}E\left[U_m(f)U_{m+k}^*(f)\right]\right)\right\} \tag{7}$$

Where:

$\text{Real}(\,.\,)$: Real-part of the expression between brackets.

Note: Expression (7) is well suitable for RPWM signals with randomized switching period [4-9].

III.1. Buck converter

In the per unit system, the input voltage E is equal to unity, thus from expression (2) the output voltage v is equal to the switching function u, (Fig.2.a). Fourier transform of such a pulse signal during the switching period T_m is:

$$U_m(f)=\frac{1}{\pi f}e^{-j2\pi f\delta_m T_m}e^{-j\pi f dT_m}\sin(\pi f dT_m)e^{-j2\pi f t_m} \quad (8)$$

Where: $\delta_m=\beta_m(1-d)$

Similarly, the complex conjugate $U^*_{m+k}(f)$ of $U_{m+k}(f)$ is:

$$U^*_{m+k}(f)=\frac{1}{\pi f}e^{j2\pi f\delta_{m+k}T_{m+k}}e^{j\pi f dT_{m+k}}\sin(\pi f dT_{m+k})e^{j2\pi f t_{m+k}} \quad (9)$$

Where: $\delta_{m+k}=\beta_{m+k}(1-d)$

Replacing $U_m(f)$ and $U^*_{m+k}(f)$ by their expressions (8) and (9) in (7) we obtain:

$$W(f)=\frac{1}{T}\left\{\begin{array}{l}E\left[\left|U_{0,m}(f)\right|^2\right]\\ +2\text{Real}\left(\sum_{k=1}^{\infty}E\left[U_{0,m}(f)U^*_{0,m+k}(f)e^{j2\pi f(t_{m+k}-t_m)}\right]\right)\end{array}\right\} \quad (10)$$

Where:

$$U_{0,m}(f)=\frac{1}{\pi f}e^{-j2\pi f\beta_m(1-d)T_m}e^{-j\pi f dT_m}\sin(\pi f dT_m) \quad (11)$$

And:

$$U^*_{0,m+k}(f)=\frac{1}{\pi f}e^{j2\pi f\beta_{m+k}(1-d)T_{m+k}}e^{j\pi f dT_{m+k}}\sin(\pi f dT_{m+k}) \quad (12)$$

Knowing that the lag time between the m^{th} and the m^{th+k} switching periods (Fig. 2) is:

$$t_{m+k}-t_m=T_m+(t_{m+k}-t_{m+1})=T_m+\sum_{l=m+1}^{m+k-1}T_l=T_m+\gamma_k \quad (13)$$

The general expression (10) becomes:

$$W(f)=\frac{1}{T}\left\{\begin{array}{l}E_{T_m}\left[\left|U_{0,m}(f)\right|^2\right]\\ +2\text{Real}\left(\sum_{k=1}^{\infty}\left(\begin{array}{l}E_{T_m,\beta_m}\left[U_{0,m}(f)e^{j\pi f T_m}\right]\\ \times E_{T_{m+k},\beta_{m+k}}\left[U^*_{0,m+k}(f)\right]\times E_T\left[e^{j2\pi f\gamma_k}\right]\end{array}\right)\right)\end{array}\right\} \quad (14)$$

Where [12]: $E_T\left[e^{j2\pi f\gamma_k}\right]=\left(E_T\left[e^{j2\pi f T}\right]\right)^{k-1}$

Finally, the infinite series over the coefficient k leads to the following general expression of the PSD:

$$W(f)=\frac{1}{T}\left\{E_T\left[\left|U(f)\right|^2\right]+2\text{Real}\frac{E_{T,\beta}\left[U(f)e^{j2\pi f T}\right]E_{T,\beta}\left[U^*(f)\right]}{1-E_T\left[e^{j2\pi f T}\right]}\right\} \quad (15)$$

Where: $U(f)=\frac{1}{\pi f}e^{-j2\pi f\beta(1-d)T}e^{-j\pi f dT}\sin(\pi f dT)$

A. Particular case of RCFM scheme

This scheme is obtained by using a carrier c with fixed fall time report ($\beta=0$) and randomized period T (Fig.2), thus from expression (15), the resulting PSD expression is:

$$W(f)=\frac{1}{T}\left\{E_T\left[\left|U(f)\right|^2\right]+2\text{Real}\left(\frac{E_T\left[U(f)e^{j2\pi f T}\right]E_T\left[U^*(f)\right]}{1-E_T\left[e^{j2\pi f T}\right]}\right)\right\} \quad (16)$$

B. Particular case of RPPM scheme

This scheme is obtained by using a carrier with fixed period T and randomized fall time report β, the resulting PSD expression is:

$$W(f)=\frac{1}{T}\left\{E_\beta\left[\left|U(f)\right|^2\right]+2\text{Real}\left(\frac{E_\beta\left[U(f)\right]E_\beta\left[U^*(f)\right]e^{j2\pi f T}}{1-e^{j2\pi f T}}\right)\right\} \quad (17)$$

For this scheme, at the multiples of the switching frequency $\left(f_k=\frac{k}{T},\ k=0,1,...\right)$, the denominator of expression (17) becomes $\left(1-e^{j2k\pi}=0\right)$, and the PSD (in Volt^2 per Hertz: v^2/Hz), has discrete components with infinite amplitudes. Thus, it is well suitable to decompose the general expression (6) of PSD into two terms: a continuous term (continuous PSD) and a discrete one (power harmonics), [4, 5] (see appendix):

$$W(f)=\frac{1}{T}\left\{\underbrace{E_\beta\left[\left|U(f)\right|^2\right]-\left|E_\beta\left[U(f)\right]\right|^2}_{\text{continuous term}}+\underbrace{\frac{1}{T}\left|E_\beta\left[U(f)\right]\right|^2\sum_{k=-\infty}^{+\infty}\delta\left(f-\frac{k}{T}\right)}_{\text{discrete term}}\right\} \quad (18)$$

III.2. Full bridge DC-DC converter

In the per unit system, E is equal to unity and the output voltage v of (Fig.2.b) is equal to the switching function $u=u_a-u_b$. Thus during the switching period T_m, Fourier transform of per unit output voltage $U_m(f)$ of Fig.2.b, can be expressed as follows:

$$U_m(f)=U_{a,m}(f)-U_{b,m}(f) \quad (19)$$

The switching functions $U_{a,m}(f)$ and $U_{b,m}(f)$ are given by:

$$U_{a,m}(f)=\frac{1}{\pi f}e^{-j\pi f d_a T_m}\sin(\pi f d_a T_m)e^{-j2\pi f\beta_m(1-d_a)T_m}e^{-j2\pi f t_m} \quad (20)$$

$$U_{b,m}(f) = \frac{1}{\pi f} e^{-j\pi f d_b T_m} \sin(\pi f d_b T_m) e^{-j2\pi f \beta_m (1-d_b) T_m} e^{-j2\pi f t_m} \qquad (21)$$

In a similar way to the buck converter, a closed form of the voltage PSD is set for the full bridge converter as follows:

$$W(f) = \frac{1}{T}\left\{ \begin{array}{l} E_{T,\beta}\left[|U(f)|^2\right] \\ + 2\text{Real}\,\dfrac{E_{T,\beta}\left[U(f)e^{j2\pi fT}\right] E_{T,\beta}\left[U^*(f)\right]}{1 - E_T\left[e^{j2\pi fT}\right]} \end{array} \right\} \qquad (22)$$

Where:

$$U(f) = \frac{1}{\pi f}\left(\begin{array}{l} e^{-j2\pi f\beta(1-d_a)T} e^{-j\pi f d_a T} \sin(\pi f d_a T) \\ - e^{-j2\pi f\beta(1-d_b)T} e^{-j\pi f d_b T} \sin(\pi f d_b T) \end{array} \right) \qquad (23)$$

$$U^*(f) = \frac{1}{\pi f}\left(\begin{array}{l} e^{j2\pi f\beta(1-d_a)T} e^{j\pi f d_a T} \sin(\pi f d_a T) \\ - e^{j2\pi f\beta(1-d_b)T} e^{j\pi f d_b T} \sin(\pi f d_b T) \end{array} \right) \qquad (24)$$

A. Particular case of RCFM scheme

The carrier has a fixed fall time report β, $(\beta = 0.5)$ and a randomized period T, which gives:

$$W(f) = \frac{1}{T}\left\{ E_T\left[|U(f)|^2\right] + 2\text{Real}\left(\frac{E_T\left[U(f)e^{j2\pi fT}\right] E_T\left[U^*(f)\right]}{1 - E_T\left[e^{j2\pi fT}\right]} \right) \right\} \qquad (25)$$

B. Particular case of RPPM scheme

The period T is fixed and the fall time report β is randomized, the PSD expression can be decomposed into a continuous term and a discrete one as follows:

$$W(f) = \frac{1}{T}\left\{ \underbrace{E_\beta\left[|U(f)|^2\right] - \left|E_\beta[U(f)]\right|^2}_{\text{continuous term}} + \underbrace{\frac{1}{T}\left|E_\beta[U(f)]\right|^2 \sum_{k=-\infty}^{+\infty} \delta\left(f - \frac{k}{T}\right)}_{\text{discrete term}} \right\} \qquad (26)$$

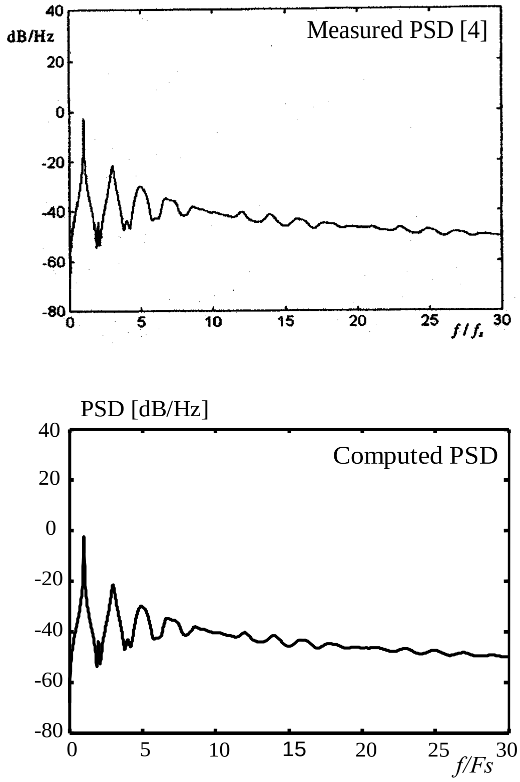
IV. VALIDATION OF PSD MODELS

IV.1. RPPM and RCFM schemes

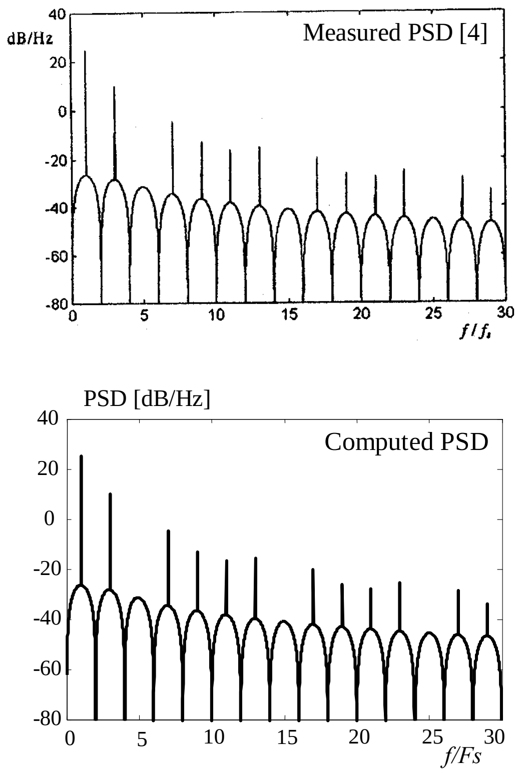
For the two converters under study, the proposed mathematical models of PSD are compared to the measure published in the literature in the same conditions.

A. Buck converter

Fig.3 shows a perfect agreement between the computed PSDs by using the proposed models (expression 16 for RCFM scheme and expression 18 for RPPM scheme) and the measure published by *K. K. Tse & all.* [4]:



a. RCFM scheme

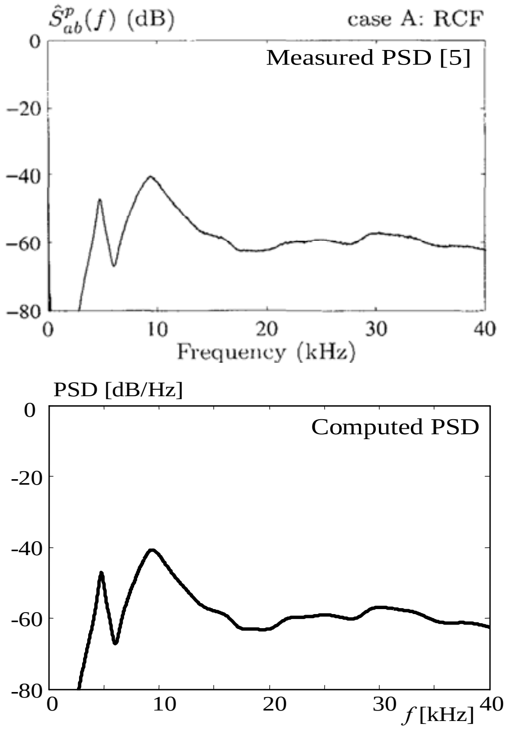


b. RPPM scheme

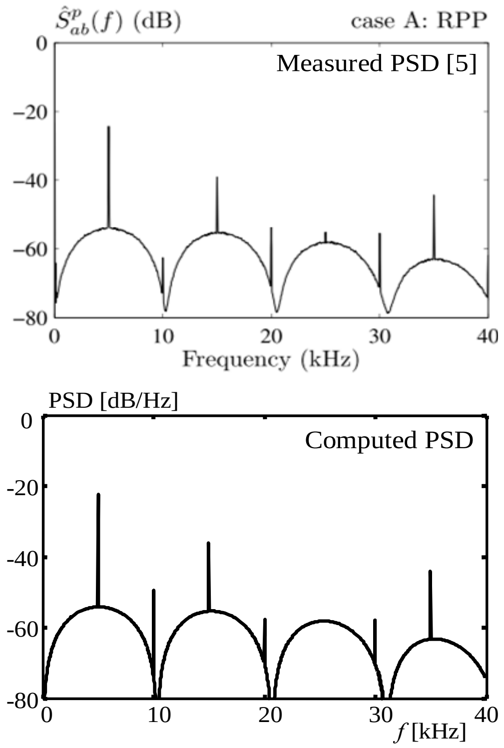
Fig.3. Computed and measured PSDs (buck converter)

B. Full bridge converter

The results of Fig.4 reveal a good agreement between the computed PSDs and the measure published by *M. M. Bech* [5], for the two schemes in the same conditions.



a. RCFM scheme



a. RPPM scheme

Fig.4. Computed and measured PSDs (full bridge converter)

As suggested in the reference [5], the duty cycles d_a and d_b of the switching signals need to be slightly corrected in order to compensate the blanking time between two complementary transistors of the same leg. However, a slight difference is noted between the calculated and the measured results because of the DC link voltage source used in the measure and the voltage drop across the power devices [5].

From Fig.3 and Fig.4, it appears that RPPM scheme is not able to spread the PSD, which contains a continuous part (power spectral density) and a discrete part (power harmonics) with important amplitudes, in the other side, RCFM scheme allows spreading completely the PSD for the two converters and reduces considerably the magnitude of the peaks, thus RCFM scheme is more advantageous than RPPM scheme.

IV.2. RCFM-RPPM scheme

For the two converters, this scheme is compared to RPPM scheme while decreasing the randomization effect of the period T (Fig.5) and to RCFM scheme while decreasing the randomization effect of parameter β (Fig.6), under the following conditions:

- Input voltage is: $E = 1$ pu.
- RCFM scheme: T is randomized in the interval:

$$\left[\overline{T}\left(1-\frac{R_T}{2}\right),\ \overline{T}\left(1+\frac{R_T}{2}\right)\right]$$

Where: $\left(\overline{T} = 1\,\text{pu}\right)$ is the statistical mean of the switching period, (i.e. period of the carrier) and R_T is the randomness level. Theoretically R_T may take any value between 0 and 2: $\left(0 \le R_T \le 2\right)$. For the buck converter, the carrier is a saw tooth $\left(\beta = 0\right)$ and for the full bridge converter, the carrier is a symmetrical triangle $\left(\beta = 0.5\right)$.

- RPPM scheme: For each converter, the period of the carrier is fixed (T = 1 pu) and the fall time report β is randomized as follows:

  a. Buck converter: β is randomized in the interval:

$$\left[0,\ R_\beta\right]$$

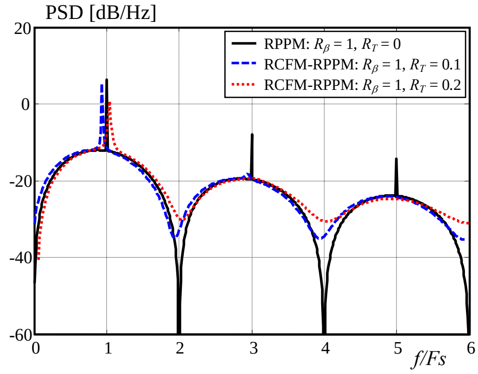
Where: $R_\beta \le 1$.

  b. Full bridge converter: β is randomized in the interval:

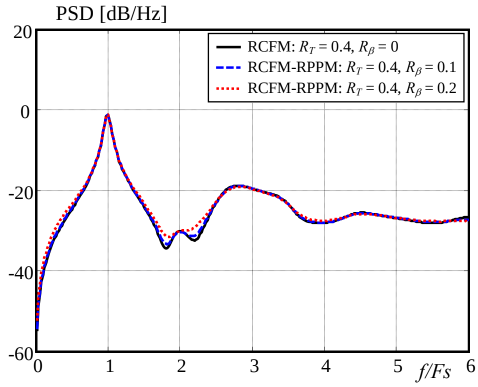
$$\left[\overline{\beta}\left(1-\frac{R_\beta}{2}\right),\ \overline{\beta}\left(1+\frac{R_\beta}{2}\right)\right]$$

Where: $\overline{\beta} = 0.5$ and $R_\beta \le 2$.

- RCFM-RPPM scheme combines the two simple RPWM schemes.
- All randomizations are performed by using uniform probability distribution function.

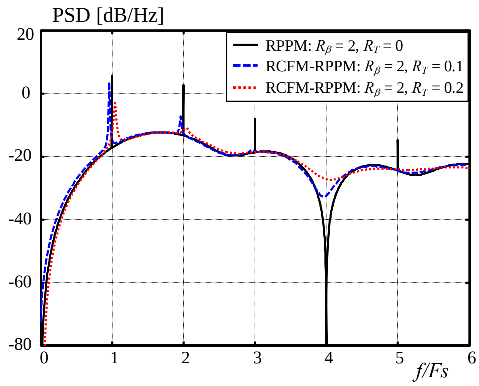


a. Comparison to RPPM

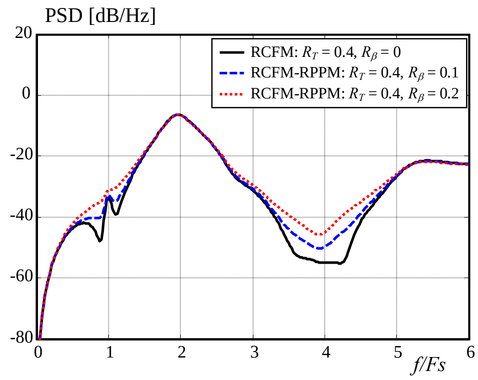


b. Comparison to RCFM

Fig.5. Comparison to simple schemes (buck converter)



a. Comparison to RPPM



b. Comparison to RCFM

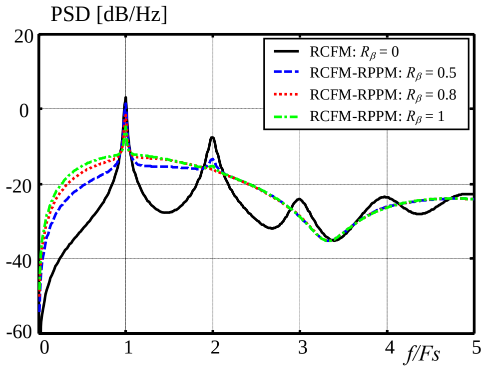
Fig.6. Comparison to simple schemes (full bridge converter)

For the two converters, Fig.5 and Fig.6 show clearly that RCFM-RPPM converges perfectly to RPPM while R_T decreases and converges to RCFM while R_β decreases, which reinforces the validity of the proposed PSD models for all schemes.

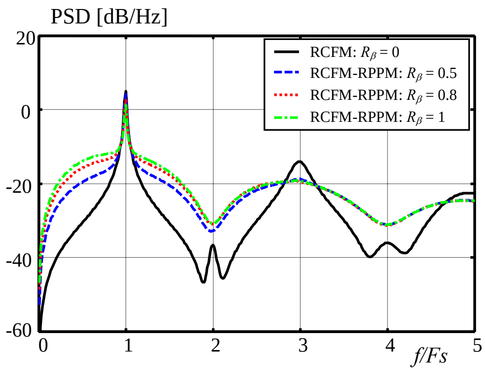
V. EMC ADVANTAGE OF THE PROPOSED RPWM SCHEME

V.1. PSD analysis

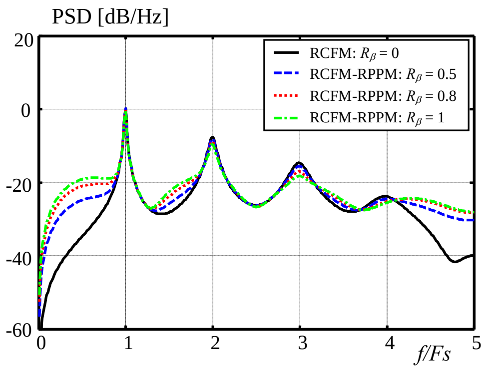
Fig.7 and Fig.8 show the PSDs of output voltage for the two converters respectively with three values of the duty cycle *d*: (*d* = 0.3, *d* = 0.5 and *d* = 0.8). In order to show the EMC advantage of the proposed dual RPWM scheme, different values of R_β are considered and the particular case of RCFM scheme (R_T = 0.2, R_β = 0) is taken as a benchmark.



a. *d* = 0.3

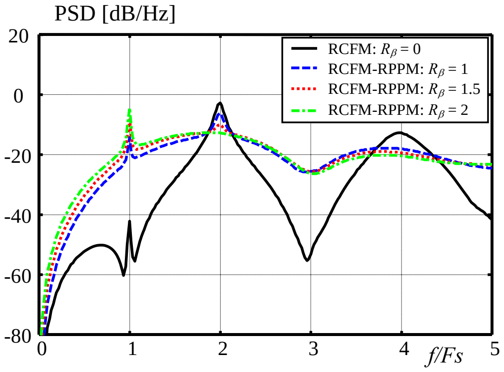


b. *d* = 0.5

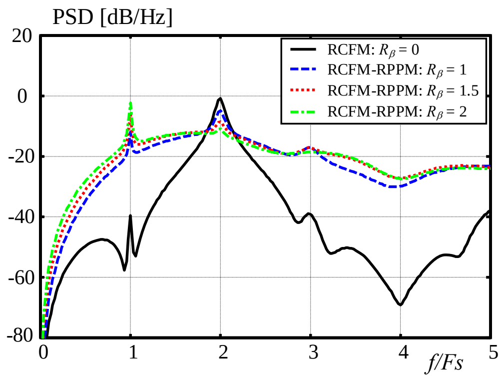


c. *d* = 0.8

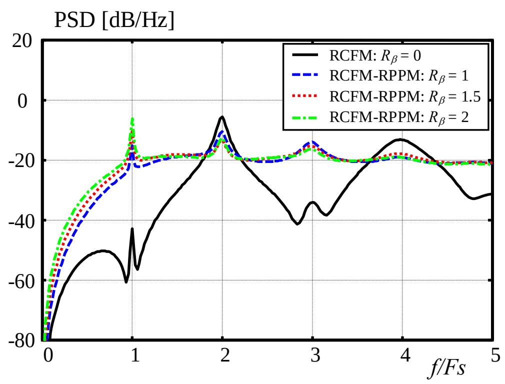
Fig.7. PSD of the output voltage (buck converter)



a. $d = 0.3$



b. $d = 0.5$



c. $d = 0.8$

Fig.8. PSD of the output voltage (full bridge converter)

For the buck converter, the randomization of β (Fig.7) adds an important spread to the PSD and a reduction of the peaks; these two advantages are more important while increasing the randomness level R_β. However, the effect of β randomization decreases for high values of the duty cycle d (Fig.7.b and Fig.7.c); indeed, a randomization of β in the interval $[0,\ R_\beta]$ gives a randomized delay report δ in the interval $[0,\ R_\beta(1-d)]$ and the upper limit $\delta_{\max} = R_\beta(1-d)$ decreases considerably while the duty cycle d rises.

The full bridge requires two switching functions u_a and u_b with duty cycles d_a and d_b satisfying the condition ($d_a + d_b = 1$), thus, the randomization of β gives two random delay reports (δ_a and δ_b) which add an important spread to the PSD for all values of d_a and d_b (Fig.8). However, we notice that for important values of R_β, $(R_\beta > 1.5)$, the peak at $(f = F_s)$ becomes important. For this reason we propose a randomness level $(1 < R_\beta < 1.5)$ in order to compromise the amplitudes of the two peaks at the frequencies: $(f = F_s \text{ and } f = 2F_s)$.

Finally, we notice that for the two converters the proposed RCFM-RPPM scheme adds an important spread to the PSD and a decrease of the peaks (Fig.7 and Fig.8), that is the desired EMC advantage.

V.2. FFT analysis

In order to confirm the results obtained by the PSD analysis, we present in this section an FFT analysis of output voltage, based on some simulation results for the two converters, (Fig.9 and Fig.10). We notice that the case of deterministic PWM (DPWM) is taken as a benchmark. The simulations are performed under the following conditions:

- Input voltage: $E = 150$ v.
- Reference signals: for the buck converter, $d = 0.5$ and for the full bridge converter, $d_a = 0.75$ and $d_b = 0.25$.
- Parameters of the carrier (T and β):

1. DPWM (fixed T and β): for the two converter ($T = \frac{1}{F_s}$, $F_s = 1800$ Hz), for the buck converter ($\beta = 0$) and for the full bridge converter ($\beta = 0.5$).
2. RPPM (fixed T and randomized β): for the two converters ($T = \frac{1}{F_s}$, $F_s = 1800$ Hz), for the buck converter β is randomized in the interval $[0, R_\beta]$ with $R_\beta = 0.9$ and for the full bridge converter β is randomized in the interval:

$$\left[\overline{\beta}\left(1-\frac{R_\beta}{2}\right),\ \overline{\beta}\left(1+\frac{R_\beta}{2}\right)\right]$$

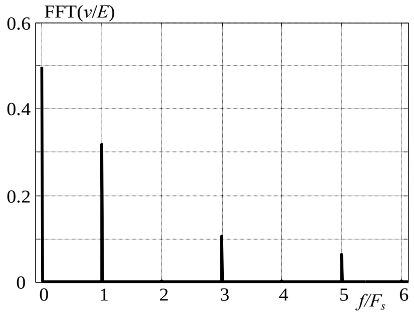
Where $\overline{\beta} = 0.5$ and $R_\beta = 1.8$.

RCFM (randomized T and fixed β): β is fixed, (for the buck converter $\beta = 0$ and for the full bridge $\beta = 0.5$) and T is randomized for the two converters in the interval:

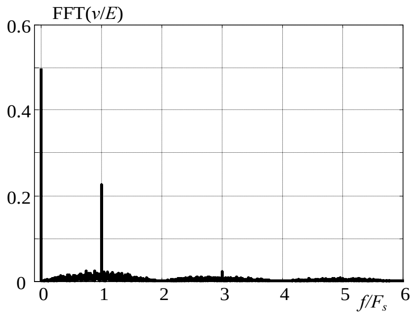
$$\left[\overline{T}\left(1-\frac{R_T}{2}\right),\ \overline{T}\left(1+\frac{R_T}{2}\right)\right]$$

Where: ($\overline{T} = \frac{1}{F_s}$, $F_s = 1800$ Hz and $R_T = 0.2$),

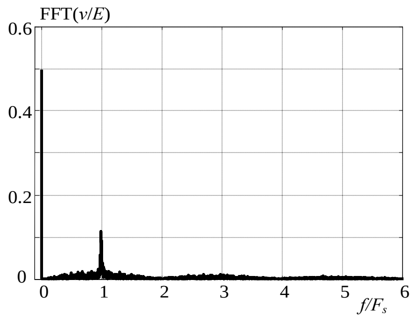
3. RCFM-RPPM: T and β are simultaneously randomized in the same way that the simple schemes.



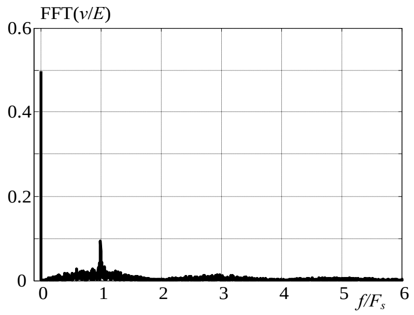
a. DPWM



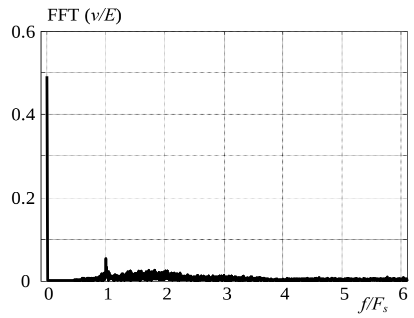
b. RPPM ($R_\beta = 0.9$)



c. RCFM ($R_T = 0.2$)

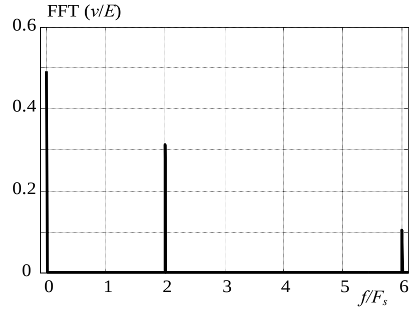


d. RCFM-RPPM ($R_T = 0.2$, $R_\beta = 0.6$)

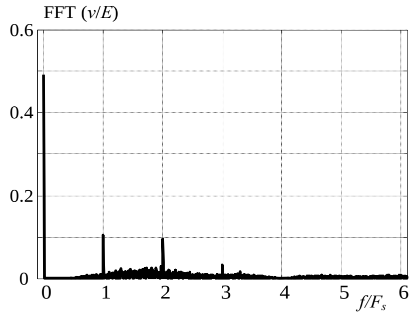


e. RCFM-RPPM ($R_T = 0.2$, $R_\beta = 0.9$)

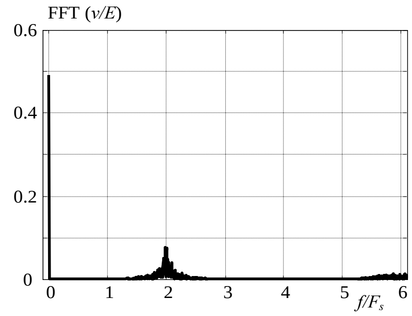
Fig.9. Spectra of output voltage (buck converter)



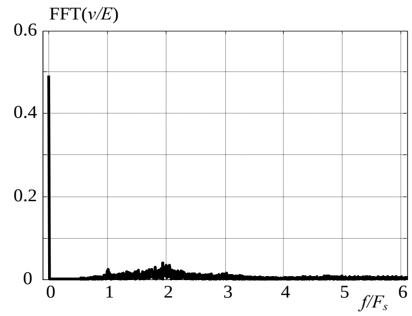
a. DPWM



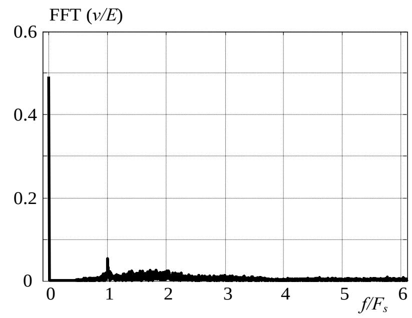
b. RPPM ($R_\beta = 1.8$)



c. RCFM ($R_T = 0.2$)



d. RCFM-RPPM ($R_T = 0.2$, $R_\beta = 1.2$)



e. RCFM-RPPM ($R_T = 0.2$, $R_\beta = 1.8$)

Fig.10. Spectra of output voltage (full bridge converter)

The spectra of Fig.9 and Fig.10 are in conformity with the obtained PSDs for all schemes:

- DPWM: discrete spectrum with important magnitude.
- RPPM: the spectrum contains a continuous part (noise) and a discrete part (harmonics); the amplitude of the discrete part is considerably reduced compared to DPWM.
- RCFM: the spectrum is completely spread into a continuous noise with relative important amplitude around F_s for the buck converter and around $2F_s$ for the full bridge converter. This scheme is more advantageous than RPPM.
- RCFM-RPPM: more spread spectrum with a reduction of the amplitude. For the full bridge converter, important values of R_β, (R_β =1.8), leads to a decrease of the amplitude around (f = $2F_s$) and an increase around (f = F_s) (Fig.10.e). As proposed for the PSD, by using R_β = 1.2, we obtain a compromise between the spectrum amplitudes at (f = F_s) and at (f = $2F_s$), (Fig.10.d).

VI. Concluding Remarks

The purpose of this work is the reduction of conducted Electromagnetic perturbations by using RPWM technique in DC-DC voltage converters. For this purpose, a dual RPWM scheme based on a triangular carrier with two randomized parameters is proposed for the buck converter and the full bridge converter. Then, a mathematical model of the PSD of the output voltage is developed and validated for the two converters. The proposed model of PSD is based directly on the randomized parameters of the carrier, which allows the treatment of the two converters in the same way. The PSD analysis shows clearly the EMC advantage of the proposed dual RPWM scheme compared to the simple RPWM schemes. Finally the FFT analysis of the voltage agrees with the PSD analysis and confirms the EMC advantage of the proposed scheme.

Appendix: Derivation of expression (18) of output voltage PSD for RPPM scheme

For the particular case of RPPM scheme, the switching period T is fixed, thus the Fourier transform (8) becomes:

$$U_m(f) = U_{0,m}(f)\, e^{-j2\pi f t_m} \quad \text{(A1)}$$

Where:

$$U_{0,m}(f) = \frac{1}{\pi f} e^{-j2\pi f \beta_m (1-d)T} e^{-j\pi f dT} \sin(\pi f dT) \quad \text{(A.2)}$$

Replacing $U_m(f)$ and $U^*_{m+k}(f)$ in the general expression (6) of the PSD, we obtain:

$$W(f) = \frac{1}{T} \sum_{k=-\infty}^{+\infty} E\left[U_{0,m}(f) U^*_{0,m+k}(f)\right] e^{j2\pi f kT} \quad \text{(A.3)}$$

In the expression (A.3), the random parameters are β_m and β_{m+k} thus the expectation of the product $E\left[U_{0,m}(f)U^*_{0,m+k}(f)\right]$ can be developed into a product of expectations $E\left[U_{0,m}(f)\right]E\left[U^*_{0,m+k}(f)\right]$ while the particular case (k = 0) is treated separately as follows [4]:

$$W(f) = \frac{1}{T}\left(\sum_{k=-\infty}^{+\infty} E_{\beta_m}\left[U_{0,m}(f)\right] E_{\beta_{m+k}}\left[U^*_{0,m+k}(f)\right] e^{j2\pi f kT} \right) + \frac{1}{T}\left(E_{\beta_m}\left[U_{0,m}(f) U^*_{0,m}(f)\right] - E_{\beta_m}\left[U_{0,m}(f)\right] E_{\beta_m}\left[U^*_{0,m}(f)\right] \right) \quad \text{(A.4)}$$

Knowing that [12]: $$\sum_{k=-\infty}^{+\infty} e^{j2\pi f kT} = \frac{1}{T}\sum_{k=-\infty}^{+\infty} \delta\left(f - \frac{k}{T}\right)$$

The general expression of the PSD becomes:

$$W(f) = \frac{1}{T}\left\{ E_\beta\left[\left|U(f)\right|^2\right] - \left|E_\beta\left[U(f)\right]\right|^2 + \frac{1}{T}\left|E_\beta\left[U(f)\right]\right|^2 \sum_{k=-\infty}^{+\infty} \delta\left(f - \frac{k}{T}\right) \right\} \quad \text{(A.5)}$$

Where: $U(f) = \frac{1}{\pi f} e^{-j2\pi f\beta(1-d)T} e^{-j\pi f dT} \sin(\pi f dT)$

References

[1] K El Khamlichi Drissi, P. C. K. Luck, B. Wang and J. Fontaine: *A Novel Dual Randomization PWM Scheme for Power Converters*, IEEE, proceedings of PESC'03, Vol. 2, pp. 480-484, June 2003.

[2] Trzynadlowski A.M., Zhiqiang Wang, Nagashima J.M., Stancu C., Zelechowski, M.H.: *Comparative investigation of PWM techniques for a new drive for electric vehicles*, IEEE, Trans. On Industry Applications, Vol. 39, Issue 5, pp. 1396-1403, Sept.-Oct. 2003.

[3] Trzynadlowski A.M., Borisov K., Yuan Li, Ling Q., Z. W.: *Mitigation of electro-magnetic interference and acoustic noise in vehicular drives by random pulse width modulation*, IEEE, proc. of Power Electron. in Transportation, pp.67 – 71, Oct. 04.

[4] K. K. Tse, Henry Shu-hung Chung, S. Y. Hui and H. C. So: *A Comparative Investigation on the Use of Random Modulation Schemes for DC/DC Converters*, IEEE Trans. On Industrial Electronics, Vol. 47, N0. 2, pp. 253-263, April 2000.

[5] Bech M. M.: "Random pulse - width modulation techniques for power electronic converters", Ph.D. Thesis, Aalborg University, Aalborg East, Denmark, 2000.

[6] R. L. Kirlin, M. M. Bech and A. M. Trzynadlowski: *Analysis of Power and Power Spectral Density in PWM Inverters with Randomized Switching Frequency*, IEEE, Trans. On Industrial Electron. Vol. 49, N0. 2, pp. 486-499, April 2002.

[7] M. Melit, N. Boudjerda, B. Nekhoul, K. El Khamlichi Drissi and K. Kerroum: *Random Modulation for Reducing Conducted perturbations in DC-DC Converters*, proceedings of EMC Europe'04, Eindhoven, Netherlands, pp. 650-655, Sept. 2004.

[8] N. Boudjerda, M. Melit, B. Nekhoul, K. El Khamlichi Drissi and K. Kerroum: *Reduction of Conducted Perturbations in DC-AC Converters by a Dual Randomization of Hybrid Space Vector Modulation*, International Review of Electrical Engineering, Vol. 1, N0 1, pp.154-161, March-April 2006.

[9] N. Boudjerda, M. Melit, B. Nekhoul, K. El Khamlichi Drissi and K. Kerroum: *Spread Spectrum in DC-DC Full Bridge Voltage Converter by a Dual Randomized PWM Scheme*, proc. of EMC Europe'08, Hamburg, Germany, pp. 181-186, Sept 2008.

[10] Kuisma, M., Rauma, K., Silventoinen, P.: *Using Switching Function in Preliminary EMI-analysis of a Switching Power Supply*, IEEE, proceedings of PESC'05, pp. 994-998, Sept. 2005.

[11] Andrzej M. Trzynadlowski: *Active Attenuation of Electromagnetic Noise in an Inverter-Fed Automotive Electric Drive System*, IEEE Trans. On Power Electronics, Vol. 21, N0. 3, pp. 693-700, May 2006.

[12] C. R. Paul: "Introduction to Electromagnetic Compatibility", Wiley-Interscience, 1992.

[13] D. Middletton: "Introduction to Statistical Communication Theory", IEEE Press, 1996.

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